Exact distribution of excursions height

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Introduction

The goal of this part of the package is to calculate the theoretical probability that the i-th excursion reach the threshold score a, for a Markov chain with a known transition matrix and a given score function. The main function for this objective is `proba_theoretical_ith_excursion_markov()`.

This section of the localScore package is the direct implementation of the pre publication: "Exact distribution of excursion heights of the Lindley process in a Markovian model" written by Carlos Cortés Rojas, Simona Grusea and Sabine Mercier.

Computation method

The main goal of this function is to calculate the probability that the first excursion of the Lindley sequence associated to the studied sequence is greater or equal to a, conditionally to α a potential beginning of this sequence. It also computes the transition probability matrix of the beginnings of excursions: `matrix_M`.

The product of this matrix, elevated to the power $i-1$, with the first vector of the probabilities conditionally to α, gives the probabilities conditionally to α for the i-th excursion. To return the global probability, the function multiplies the conditional vector with the distribution of the first letter of the sequence.

Definition of a mathematical excursion

The number of an excursion is given by the mathematical definition of Karlin and Altschul (1990). Its corresponds to the number of record times of the Lindley sequence associated to the score sequence $(X_k)_{1\leq k\leq n}$ and recursively defined as follows:

$$T_0 := 0 \qquad \text{and} \qquad T_{(k+1)} := \inf\{i \geq T_k + 1, \sum_{j=T_k+1}^{i} X_j < 0\}\ .$$

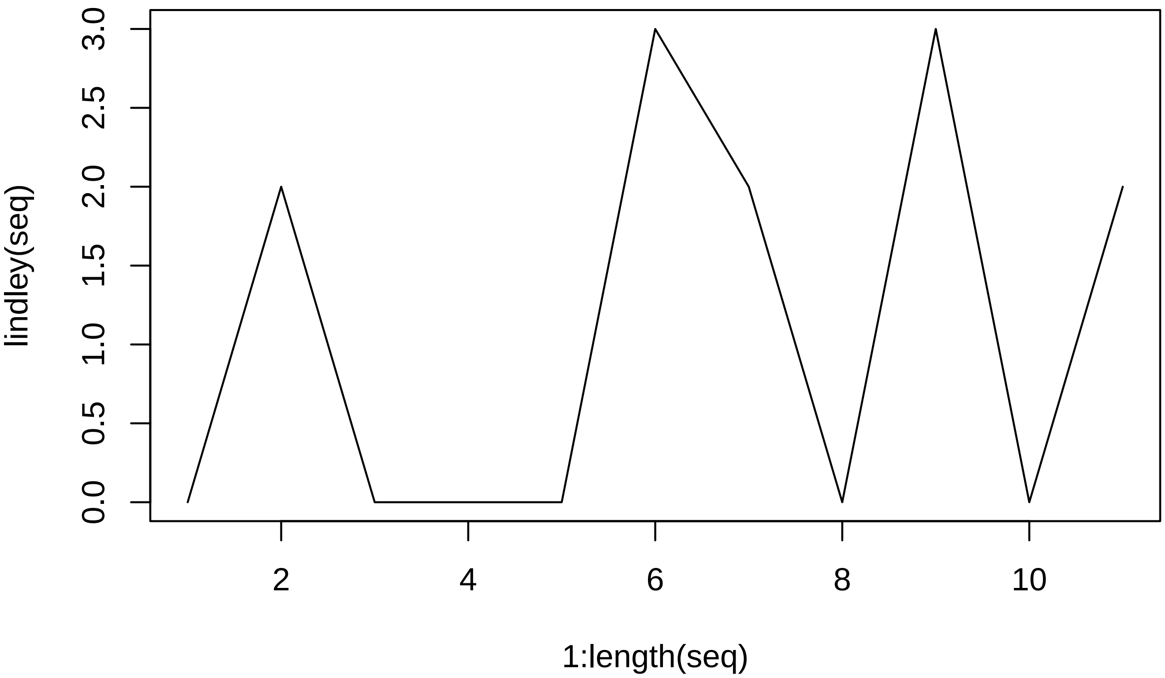
Please note that the sum $\sum_{j=T_k+1}^{i} X_j$ must be non positive. If it is equal to zero, it will mathematically be still the same Lindley excursion.

$$(A_k)_k \quad \text{with} \quad T_{(i-1)} + 1 \leq k < T_i$$

is called the i-th positive excursion of the sequence in the term of Karlin and Altschul. This mathematical definition includes the index with a Linldey score equal to 0, whereas visually we “see” the excursion beginning at the following index.

The mathematical number of an excursion must be distinguished from the one of the corresponding visible excursion because of the possible appearance of flat excursions. See below for an example.

```
seq <- c(-1,2,-3,-4,-1,3,-1,-2,3,-4,2)
lindley(seq)
#>  [1] 0 2 0 0 0 3 2 0 3 0 2
plot(1:length(seq), lindley(seq), type = 'l')
```



```
localScoreC(seq)
#> $localScore
#> value begin   end
#>     3     6     6
#>
#> $suboptimalSegmentScores
```

```
#>   value begin end
#> 1     2     2   2
#> 2     3     6   6
#> 3     3     9   9
#> 4     2    11  11
#> 
#> $RecordTime
#> [1]  0  1  3  4  5 10
```

We can "see" three positive excursions, with top at index $\{2; 6; 9\}$; plus one last not achieved at index $\{11\}$. Their exists also five flats excursions: at index $\{1; 3; 4; 5; 10\}$. Note also that the Lindley excursion 6-10 reach 0 at index 8 without going in non positive values, so it is still the same excursion.

In summary, the excursions list is: $\{1; 2-3; 4; 5; 6-10; 11\}$. The following function gives the record time of the mathematical excursion that must be used to compute the statistical significance. Note that we choose to omit the conventional first record time which is always 0.

```
recordTimes(seq)
#> [1]  1  3  4  5 10
```

So the sub optimal excursion values given by `$suboptimalSegmentScores` 2 3 3 2 are achieved by the excursion number 2, 5 (twice) and 6. At this point, we wish to lighten a difference between the notion of "excursion" defined mathematically by Karlin and Dembo (1992), and the results given in `$suboptimalSegmentScores`. The segment [6-10] is an excursion in mathematical sense as the cumulative process starting at position 7 never hit a negative value before the 10th position. Nevetheless this process hit a null value indicating that this single excursion contain at least two positive score segments (here : at position 6 and 9). We chose to indicate in `$suboptimalSegmentScores` all positive consecutive sub-sequences, as it can be more meaningful when applied on real data.

To calculate the p-value of a given excursion, we have to know which number of excursion it is in sequential order. Notice that it will not bring any problem of making a mistake in the number of the excursion for a number exceeding 10, and a small difference for a lower number, as the sequence composed by the begin-component of each excursion, which is also a Markov chain, can reach the stationary distribution. For the first excursions, it is far more easy to recover the record time of the corresponding mathematical excursion in the list.

Toy examples

As the function calculates a theoretical probability, we don't need a sequence of scores but the transition matrix of the markov chain: Λ. It also requires a score function with: integer scores, a negative expectation and at least one possible positive score. We also need θ, an alphabet (can contains numbers) with unique values ; and i: the rank of the excursion on which we calculate the probabilities. The optional parameters are `epsilon` and `prob0`. `epsilon` is a threshold for the computation of matrix M and `prob0` is the probability distribution of the first letter of the sequence. Obviously, `theta`, the score function and `prob0` should have the same length p, and the dimension of `lambda` should be (p, p).

In this example, we want to know the probability that the second excursion reach a score greater or equal than 5 with the matrix Λ and a score function of $(-1, 1)$, given that the first score of the sequence have 50 percent chances to be -1.

```
proba_theoretical_ith_excursion_markov(a = 5,
                                       theta = c("A", "B"),
                                       lambda = matrix(c(0.5, 0.5, 0.8, 0.2),
```

```
                                                       ncol = 2, byrow = TRUE),
                                       score_function = c(-1,1),
                                       i = 2,
                                       epsilon = 1e-16,
                                       prob0 = c(0.5, 0.5))
#> $proba_q_i_geq_a
#> [1] 0.009661836
#> 
#> $P_alpha
#>           A           B 
#> 0.009661836 0.009661836
```

In this following example, we see that this is not a problem to have missing score values in the score function, and that theta can contain numbers. Initial value of `epsilon` is 1e-16, and if we don't precise the value of `prob0`, the function compute the stationary distribution of Λ, and use it for the distribution of the first letter of the sequence.

```
proba_theoretical_ith_excursion_markov(a = 5,
                                       theta = c("K", 12, 1.54),
                                       lambda = matrix(c(0.9, 0, 0.1, 0.9, 0.1, 0, 0, 0.8, 0.2),
                                                       ncol = 3, byrow = TRUE),
                                       score_function = c(-3,-1,2),
                                       i = 4)
#> $proba_q_i_geq_a
#> [1] 0.00405119
#> 
#> $P_alpha
#>           K          12        1.54 
#> 0.004054779 0.004050819 0.004022805
```

In the following example, the function takes longer as we increased the number of scores (complexity of $O(length(\theta)^3)$), but here with 20 scores, the computational time is acceptable.

```
transition_matrix <- matrix(runif(400, min = 0, max = 1), nrow = 20)
transition_matrix <- t(apply(transition_matrix, 1, function(x) x/sum(x)))
theta <- letters[1:20]
score_f <- c(-3,-1,2,-3,-1,2,-3,-1,2,-1,
              -3,-1,2,-3,-1,2,-3,-1,2,-1)
sum(stationary_distribution(transition_matrix)*score_f) #score expectation (stationary)
#> [1] -0.647538
system.time(pv1 <- proba_theoretical_ith_excursion_markov(a=5, theta, transition_matrix, score_f, i = 4
#> utilisateur     système      écoulé 
#>       4.911       0.004       4.917
pv1$proba_q_i_geq_a
#> [1] 0.07114636
```

A study case

Let us consider the protein `Seq1093` of 1093 amino acid proposed in the package `localScore` which corresponds to the Q60519.fasta in UniProt Data base.

Optimal excursion, the local score

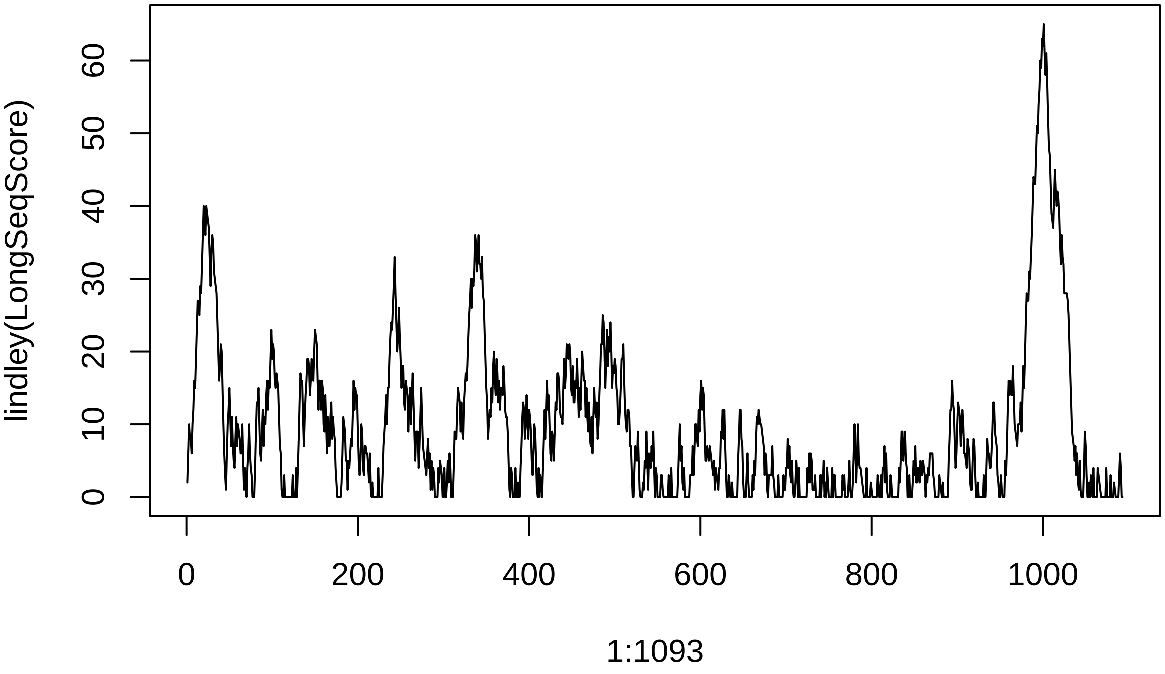
Using the hydrophobic score scale called `HydroScore`, we compute the local score, corresponding to the height of the highest excursion.

```
library(localScore)
data("Seq1093")
Seq1093
#> [1] "MVVPGPLALSLLLSSLTLLVSHLSSSQDIASESSSEQQMCTRREHPIVAFEDLKPWVFNFTYPGVRDFSQLALDPSRNQLIVGARNYLFRLSLAN
nchar(Seq1093)
#> [1] 1093
data(HydroScore)
LongSeqScore <- CharSequence2ScoreSequence(Seq1093, HydroScore)
table(LongSeqScore)
#> LongSeqScore
#>  -5  -4  -3  -2  -1   0   2   3   4   5
#>  81 222  22  82 232  85  78 103 157  31
localScoreC(LongSeqScore)
#> $localScore
#> value begin   end
#>    65   956  1001
#>
#> $suboptimalSegmentScores
#>    value begin  end
#> 1     40     1   20
#> 2     10    71   73
#> 3     23    80   99
#> 4      3   114  114
#> 5      3   124  124
#> 6      4   128  128
#> 7     23   130  150
#> 8     16   181  195
#> 9      2   217  217
#> 10     4   224  224
#> 11    33   229  243
#> 12     5   294  296
#> 13     4   301  301
#> 14     6   304  307
#> 15    36   312  337
#> 16     4   379  379
#> 17     4   384  384
#> 18     2   387  387
#> 19    14   390  397
#> 20     4   411  411
#> 21     3   413  413
#> 22    25   416  486
#> 23     9   523  527
#> 24     9   533  537
#> 25     4   548  548
#> 26     3   554  554
#> 27     3   563  563
#> 28     4   566  566
#> 29    10   574  576
#> 30    16   588  601
```

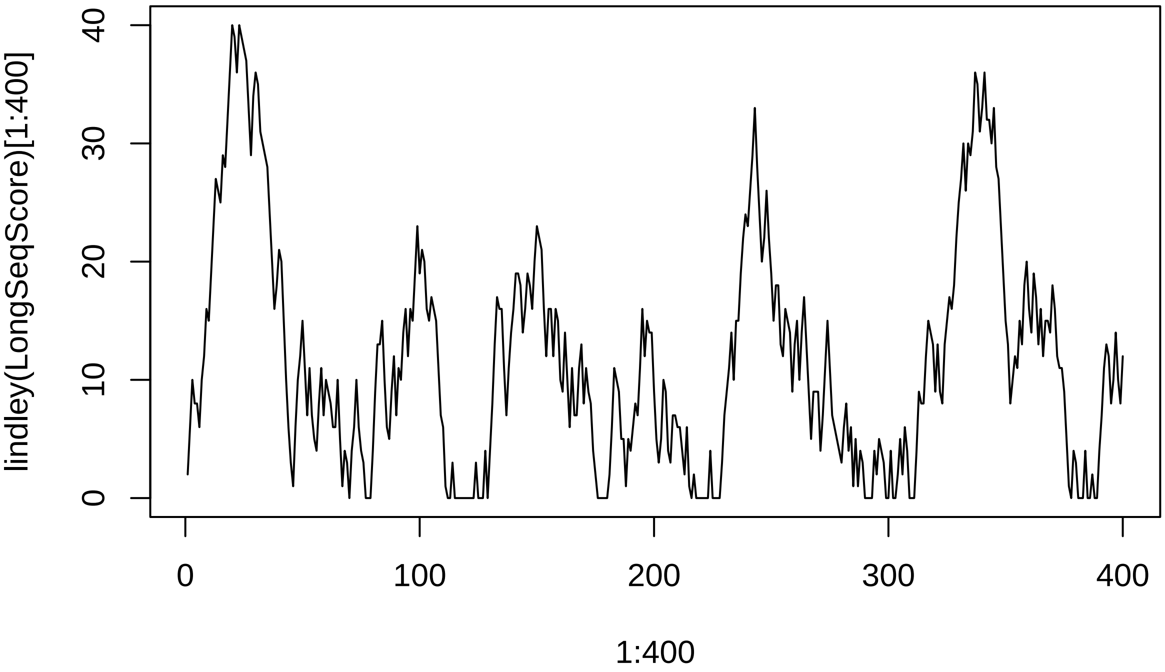
```
#> 31      3   633  633
#> 32      2   637  637
#> 33     12   644  646
#> 34      6   654  655
#> 35     12   661  668
#> 36      7   680  684
#> 37      3   691  691
#> 38      8   697  702
#> 39      5   711  712
#> 40      4   715  715
#> 41      6   725  727
#> 42      3   740  740
#> 43      5   742  744
#> 44      2   746  746
#> 45      4   748  748
#> 46      4   754  754
#> 47      3   756  756
#> 48      3   766  766
#> 49      5   773  774
#> 50     10   778  780
#> 51      4   794  794
#> 52      2   799  799
#> 53      3   807  807
#> 54      3   811  811
#> 55      7   813  815
#> 56      3   822  822
#> 57      9   832  835
#> 58      3   844  844
#> 59      7   848  851
#> 60      6   864  868
#> 61      3   879  879
#> 62      2   883  883
#> 63     16   890  894
#> 64      2   924  924
#> 65      3   931  931
#> 66     13   934  942
#> 67      3   951  951
#> 68     65   956 1001
#> 69      9  1048 1049
#> 70      2  1054 1054
#> 71      3  1056 1056
#> 72      4  1059 1059
#> 73      4  1064 1064
#> 74      4  1074 1074
#> 75      3  1079 1079
#> 76      2  1083 1083
#> 77      6  1089 1090
#>
#> $RecordTime
#>   [1]    0   70   77   78   79  112  113  115  116  117  119  120  121  122  123
#>  [16]  125  126  127  129  176  177  178  179  180  218  219  220  221  222  223
#>  [31]  226  227  228  290  291  293  299  300  303  310  311  378  381  382  383
#>  [46]  386  388  389  412  415  521  522  531  532  550  551  558  559  560  561
```

```
#>  [61]  562  564  565  569  570  571  573  583  584  585  586  587  631  632  635
#>  [76]  636  639  640  643  651  652  653  657  658  659  660  679  687  688  689
#>  [91]  690  692  693  695  696  709  710  714  716  717  718  719  720  721  722
#> [106]  723  724  735  736  737  738  739  741  750  751  752  758  759  760  761
#> [121]  762  763  764  765  769  771  777  791  792  793  798  803  804  805  806
#> [136]  809  810  812  819  820  821  824  825  826  827  828  829  843  845  846
#> [151]  847  863  874  875  876  877  878  882  885  886  888  889  922  925  926
#> [166]  927  928  929  930  932  933  949  950  954  955 1045 1046 1047 1052 1053
#> [181] 1055 1061 1062 1063 1069 1070 1071 1072 1073 1076 1077 1078 1080 1081 1082
#> [196] 1085 1087 1088 1093

head(sort(localScoreC(LongSeqScore)$suboptimalSegmentScores[,1], decreasing = TRUE))
#> [1] 65 40 36 33 25 23
plot(1:1093,lindley(LongSeqScore), type = 'l')
```



```
plot(1:400,lindley(LongSeqScore)[1:400], type = 'l')
```



We get: local score = 65. Let us compute its p-value in a I.I.D. model.

```
LS <- localScoreC(LongSeqScore)$localScore["value"]
prob1 <- scoreSequences2probabilityVector(list(LongSeqScore))
prob1
#>          -5          -4          -3          -2          -1           0           1
#> 0.07410796 0.20311070 0.02012809 0.07502287 0.21225984 0.07776761 0.00000000
#>           2           3           4           5
#> 0.07136322 0.09423605 0.14364135 0.02836231
daudin(local_score = LS, sequence_length = length(LongSeqScore),
       score_probabilities = prob1,
       sequence_min = min(LongSeqScore),
       sequence_max = max(LongSeqScore))
#> [1] 0.07231933
```

In an independent model the p-value equals 7.2%, that is non significant using the nominal level 5%.

Let us compute it in a markovian model. We need the transition matrix.

```
tmpMarkovParameters <- sequences2transmatrix(LongSeqScore)
lambda <- tmpMarkovParameters$transition_matrix
apply(lambda, 1, sum) #to check stochasticity
#> -5 -4 -3 -2 -1  0  2  3  4  5
#>  1  1  1  1  1  1  1  1  1  1
prob0 <- stationary_distribution(lambda)
print(prob0)
#>  [1] 0.07415553 0.20331534 0.02016219 0.07512607 0.21241680 0.07789931
#>  [7] 0.07052191 0.09425453 0.14375820 0.02839011
score_values <- tmpMarkovParameters$score_value
print(score_values)
#>  [1] -5 -4 -3 -2 -1  0  2  3  4  5
```

Notice that the score value 1 is not present in the sequence.

```
exact_mc(LS, lambda, sequence_length = length(LongSeqScore), score_values = score_values)
#> [1] 0.07695456
```

The p-value in a Markovian model, equal to 7.7% is very similar in this case with the one of the independent model.

Sub optimal excursions

Let us consider a study on the first and the last sub optimal excursions.

```
subOptSegment <- localScoreC(LongSeqScore)$suboptimalSegmentScores
o <- order(subOptSegment[,1], decreasing = TRUE)
subOptSegment <- subOptSegment[o,] # reordering segments by decreasing score values
print(subOptSegment)
#>    value begin  end
#> 68    65   956 1001
#> 1     40     1   20
#> 15    36   312  337
#> 11    33   229  243
#> 22    25   416  486
#> 3     23    80   99
#> 7     23   130  150
#> 8     16   181  195
#> 30    16   588  601
#> 63    16   890  894
#> 19    14   390  397
#> 66    13   934  942
#> 33    12   644  646
#> 35    12   661  668
#> 2     10    71   73
#> 29    10   574  576
#> 50    10   778  780
#> 23     9   523  527
#> 24     9   533  537
#> 57     9   832  835
#> 69     9  1048 1049
#> 38     8   697  702
#> 36     7   680  684
#> 55     7   813  815
#> 59     7   848  851
#> 14     6   304  307
#> 34     6   654  655
#> 41     6   725  727
#> 60     6   864  868
#> 77     6  1089 1090
#> 12     5   294  296
#> 39     5   711  712
#> 43     5   742  744
#> 49     5   773  774
#> 6      4   128  128
#> 10     4   224  224
```

```
#> 13     4   301  301
#> 16     4   379  379
#> 17     4   384  384
#> 20     4   411  411
#> 25     4   548  548
#> 28     4   566  566
#> 40     4   715  715
#> 45     4   748  748
#> 46     4   754  754
#> 51     4   794  794
#> 72     4  1059 1059
#> 73     4  1064 1064
#> 74     4  1074 1074
#> 4      3   114  114
#> 5      3   124  124
#> 21     3   413  413
#> 26     3   554  554
#> 27     3   563  563
#> 31     3   633  633
#> 37     3   691  691
#> 42     3   740  740
#> 47     3   756  756
#> 48     3   766  766
#> 53     3   807  807
#> 54     3   811  811
#> 56     3   822  822
#> 58     3   844  844
#> 61     3   879  879
#> 65     3   931  931
#> 67     3   951  951
#> 71     3  1056 1056
#> 75     3  1079 1079
#> 9      2   217  217
#> 18     2   387  387
#> 32     2   637  637
#> 44     2   746  746
#> 52     2   799  799
#> 62     2   883  883
#> 64     2   924  924
#> 70     2  1054 1054
#> 76     2  1083 1083
```

The sub optimal scores are 40, 36 and 33 realized by the “visual” excursions number 1, 15 and 11. Such number of excursion, 11 and 15 are large enough to avoid considering the mathematical number. The first excursion is mathematically the first one (see below).

```
lindley(LongSeqScore)
#>    [1]  2  6 10  8  8  6 10 12 16 15 19 23 27 26 25 29 28 32 36 40 39 36 40 39
#>   [25] 38 37 33 29 34 36 35 31 30 29 28 24 20 16 18 21 20 15 10  6  3  1  6 10
#>   [49] 12 15 11  7 11  7  5  4  8 11  7 10  9  8  6  6 10  5  1  4  3  0  4  6
#>   [73] 10  6  4  3  0  0  0  4  9 13 13 15 10  6  5  9 12  7 11 10 14 16 12 16
#>   [97] 15 19 23 19 21 20 16 15 17 16 15 11  7  6  1  0  0  3  0  0  0  0  0  0
#>  [121]  0  0  0  3  0  0  0  4  0  4  8 13 17 16 16 11  7 11 14 16 19 19 18 14
#>  [145] 16 19 18 16 20 23 22 21 16 12 16 16 12 16 15 10  9 14 10  6 11  7  7 11
```

```
#>  [169] 13  8 11  9  8  4  2  0  0  0  0  0  2  6 11 10  9  5  5  1  5  4  6  8
#>  [193]  7 11 16 12 15 14 14  9  5  3  5 10  9  4  3  7  7  6  6  4  2  6  1  0
#>  [217]  2  0  0  0  0  0  0  4  0  0  0  0  3  7  9 11 14 10 15 15 19 22 24 23
#>  [241] 26 29 33 28 24 20 22 26 22 19 15 18 18 13 12 16 15 14  9 13 15 10 14 17
#>  [265] 13  9  5  9  9  9  4  7 11 15 11  7  6  5  4  3  6  8  4  6  1  5  1  4
#>  [289]  3  0  0  0  0  4  2  5  4  3  0  0  4  0  0  2  5  2  6  4  0  0  0  4
#>  [313]  9  8  8 12 15 14 13  9 13  9  8 13 15 17 16 18 22 25 27 30 26 30 29 31
#>  [337] 36 35 31 33 36 32 32 30 33 28 27 23 19 15 13  8 10 12 11 15 13 18 20 16
#>  [361] 14 19 17 13 16 12 15 15 14 18 16 12 11 11  9  5  1  0  4  3  0  0  0  4
#>  [385]  0  0  2  0  0  4  7 11 13 12  8 10 14 10  8 12 11  9  5  3  6 10  9  5
#>  [409]  1  0  4  0  3  2  0  4  8 12  8 12 16 12 14 10  6  5  9  8  5  9 13 12
#>  [433] 17 17 16 12 11 11 10 15 19 15 17 21 20 19 21 20 15 14 18 13 13 16 15 19
#>  [457] 15 11 15 12 16 20 18 16 16 11 15 11  9 13  8  7 11  6 11 15 12 11 13  8
#>  [481] 10 14 17 21 21 25 24 20 15 19 23 18 22 20 24 20 15 18 17 19 18 15 14 10
#>  [505] 10 12 15 19 19 21 16 12 10  9 12 12 11  7  7  3  0  0  4  7  6  5  9  5
#>  [529]  1  0  0  0  2  1  5  4  9  5  1  6  5  4  7  5  9  4  0  4  3  0  0  0
#>  [553]  0  3  3  1  0  0  0  0  0  0  3  0  0  4  0  0  0  0  0  0  0  3  7 10
#>  [577]  5  7  2  1  4  0  0  0  0  0  0  3  3  3  7  3  6 10 10  8  7 12  9 14
#>  [601] 16 12 15 14  9  5  5  7  6  5  7  6  5  4  3  5  1  4  3  2  1  4  4  9
#>  [625]  9 12  8 12  7  3  0  0  3  2  0  0  2  0  0  0  0  0  0  5  8 12 12  8
#>  [649]  7  2  0  0  0  3  6  2  0  0  0  0  3  1  5  3  8 11 10 12 11 10 10  9
#>  [673]  8  7  3  6  5  1  0  3  3  3  3  7  3  2  0  0  0  0  3  0  0  0  0  0
#>  [697]  3  1  1  4  4  8  4  7  3  2  5  1  0  0  2  5  3  0  4  0  0  0  0  0
#>  [721]  0  0  0  0  4  2  6  2  6  5  1  1  1  3  0  0  0  0  0  3  0  3  2  5
#>  [745]  0  2  0  4  2  0  0  0  0  4  0  3  3  0  0  0  0  0  0  0  0  3  1  3
#>  [769]  0  0  0  0  2  5  1  0  0  2  6 10  6  2  6 10  5  4  4  3  2  1  0  0
#>  [793]  0  4  0  0  0  0  2  1  0  0  0  0  0  0  3  2  0  0  3  0  4  4  7  2
#>  [817]  6  1  0  0  0  3  2  0  0  0  0  0  0  0  0  4  2  5  9  9  5  7  9  5
#>  [841]  4  0  0  3  0  0  0  2  5  3  7  2  2  4  3  2  5  4  3  5  4  3  0  3
#>  [865]  2  4  3  6  6  6  6  3  2  0  0  0  0  0  3  2  1  0  2  0  0  0  0  0
#>  [889]  0  5  8 12 12 16 13 12  8  4  6 10 13 12 11  7  9 12 10  6  6  5  4  8
#>  [913]  7  6  2  1  1  5  8  7  3  0  0  2  0  0  0  0  0  0  3  0  0  4  8  6
#>  [937]  6  4  4  6  9 13 13  9  8  7  3  2  0  0  3  1  0  0  0  5  3  7 12 16
#>  [961] 14 16 15 14 18 14 10  9  8  7 10 10 10 13  9 13 18 15 19 24 28 27 27 31
#>  [985] 30 33 36 40 44 43 43 47 51 50 54 56 60 59 63 62 65 61 58 61 57 52 48 47
#> [1009] 43 39 38 37 41 45 42 40 42 41 39 35 32 36 33 32 28 28 28 28 27 25 21 17
#> [1033] 13  9  8  7  5  7  3  6  2  1  5  1  0  0  0  4  9  7  3  0  0  2  0  3
#> [1057]  2  0  4  0  0  0  0  4  3  2  1  0  0  0  0  0  0  4  0  0  0  0  3  0
#> [1081]  0  0  2  1  0  0  0  0  3  6  4  0  0
recordTimes(LongSeqScore)
#>   [1]   70   77   78   79  112  113  115  116  117  119  120  121  122  123  125
#>  [16]  126  127  129  176  177  178  179  180  218  219  220  221  222  223  226
#>  [31]  227  228  290  291  293  299  300  303  310  311  378  381  382  383  386
#>  [46]  388  389  412  415  521  522  531  532  550  551  558  559  560  561  562
#>  [61]  564  565  569  570  571  573  583  584  585  586  587  631  632  635  636
#>  [76]  639  640  643  651  652  653  657  658  659  660  679  687  688  689  690
#>  [91]  692  693  695  696  709  710  714  716  717  718  719  720  721  722  723
#> [106]  724  735  736  737  738  739  741  750  751  752  758  759  760  761  762
#> [121]  763  764  765  769  771  777  791  792  793  798  803  804  805  806  809
#> [136]  810  812  819  820  821  824  825  826  827  828  829  843  845  846  847
#> [151]  863  874  875  876  877  878  882  885  886  888  889  922  925  926  927
#> [166]  928  929  930  932  933  949  950  954  955 1045 1046 1047 1052 1053 1055
#> [181] 1061 1062 1063 1069 1070 1071 1072 1073 1076 1077 1078 1080 1081 1082 1085
```

```
#> [196] 1087 1088 1093
LongSeqScore[1] > 0
#> [1] TRUE
```

Let us consider a study on the first and the last sub optimal excursions in the sequential order of the sequence.

First excursion height equal to 40; last one, the 77th one, height equal to 6.

```
subOptSegment["1",] #First excursion
#>   value begin end
#> 1    40     1  20
subOptSegment[as.character(dim(subOptSegment)[1]),] #Last excursion
#>    value begin  end
#> 77     6  1089 1090
```

Let us compute their p-values.

```
theta <- letters[1:length(score_values)] # arbitrary
score_function <- score_values           # defined earlier
a <- 40
i <- 1
system.time(pv2<-proba_theoretical_ith_excursion_markov(a, theta, lambda,
                                       score_function,i)$proba_q_i_geq_a)
#> utilisateur     système      écoulé
#>      39.004       0.014      39.050
pv2
#> [1] 0.004478564

a <- 6
i <- 77
system.time(pv3<-proba_theoretical_ith_excursion_markov(a, theta, lambda,
                                       score_function, i)$proba_q_i_geq_a)
#> utilisateur     système      écoulé
#>       1.682       0.018       1.700
pv3
#> [1] 0.1741784
```

First excursion : height equal to 40 with p-value=0.45%; Last excursion, the 77th one, height equal to 6 and p-value=17%.

The time computation of the first p-value is larger because of the larger value of a.

We can consider that from the 20th mountain we reached the stationary distribution for the beginning of excursion. We shall therefore take a lower value for i for the last excursion to evaluate the difference. Let us try $i = 20$ instead of 77.

```
a <- 6
i <- 20
system.time(pv4 <- proba_theoretical_ith_excursion_markov(a, theta,
                                       lambda,
                                       score_function,i)$proba_q_i_geq_a)
#> utilisateur     système      écoulé
#>       1.700       0.000       1.701
pv4
```

```
#> [1] 0.1741784

i <-  10
system.time(pv5 <- proba_theoretical_ith_excursion_markov(a, theta,
                                        lambda,
                                        score_function,i)$proba_q_i_geq_a)
#> utilisateur     système      écoulé
#>       1.799       0.002       1.802
pv5
#> [1] 0.1741784
```

We obtain the same value as expected even for $i = 10$.

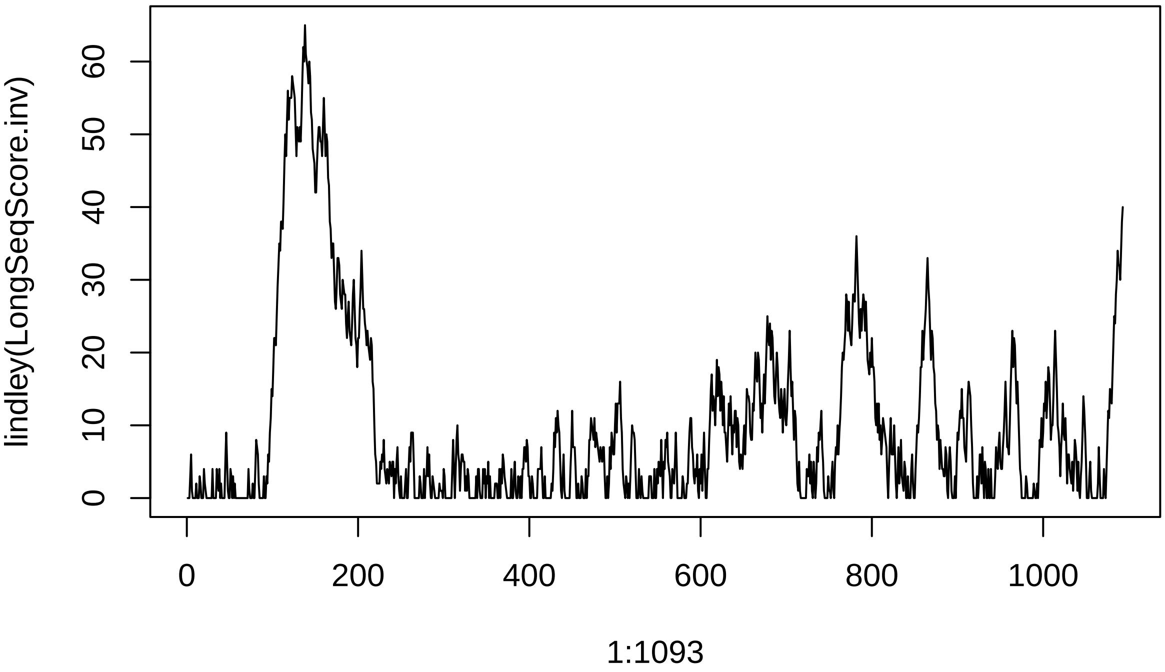
With a reverse lecture of the protein

As the lecture of a protein could be done in both direction, we also consider the reverse sequence.

```
LongSeqScore.inv <- rev(LongSeqScore)
localScoreC(LongSeqScore.inv)
#> $localScore
#> value begin   end
#>    65    93   138
#>
#> $suboptimalSegmentScores
#>    value begin  end
#> 1      6     4    5
#> 2      2    11   11
#> 3      3    15   15
#> 4      4    20   20
#> 5      4    30   30
#> 6      4    35   35
#> 7      2    40   40
#> 8      9    45   46
#> 9      4    51   51
#> 10     3    54   54
#> 11     2    56   56
#> 12     4    72   72
#> 13     2    77   77
#> 14     8    80   81
#> 15     3    90   90
#> 16    65    93  138
#> 17     7   243  246
#> 18     3   250  250
#> 19     4   255  256
#> 20     9   259  262
#> 21     3   272  272
#> 22     4   277  277
#> 23     7   279  281
#> 24     3   287  287
#> 25     2   295  295
#> 26     4   300  300
#> 27     8   310  311
```

```
#> 28     10   314  316
#> 29      3   338  338
#> 30      4   340  340
#> 31      4   346  346
#> 32      5   350  352
#> 33      3   354  354
#> 34      2   360  360
#> 35      4   365  365
#> 36      6   367  369
#> 37      4   379  379
#> 38      5   382  383
#> 39      3   387  387
#> 40      3   390  390
#> 41      8   392  397
#> 42      3   403  403
#> 43      7   410  414
#> 44      3   418  418
#> 45     12   426  433
#> 46      6   439  440
#> 47     12   448  450
#> 48      2   457  457
#> 49      3   461  461
#> 50      4   466  466
#> 51     11   468  472
#> 52      3   491  491
#> 53     16   493  506
#> 54      3   513  513
#> 55      2   516  516
#> 56     10   518  520
#> 57      4   528  528
#> 58      3   531  531
#> 59      3   540  540
#> 60      4   546  546
#> 61      8   549  554
#> 62      9   557  561
#> 63      9   567  571
#> 64      3   579  579
#> 65     11   584  588
#> 66      9   599  604
#> 67     25   608  678
#> 68      6   724  727
#> 69      5   732  732
#> 70     12   735  741
#> 71      3   749  749
#> 72      5   753  754
#> 73     36   757  782
#> 74     11   820  822
#> 75      8   830  834
#> 76      3   842  842
#> 77      6   846  847
#> 78     33   851  865
#> 79      7   890  891
#> 80      3   897  897
```

```
#> 81    16   899  913
#> 82     3   923  923
#> 83     7   925  929
#> 84     5   932  932
#> 85     4   936  936
#> 86     4   939  939
#> 87    23   944  964
#> 88     3   980  980
#> 89     2   989  989
#> 90     2   993  993
#> 91    23   995 1014
#> 92    14  1044 1047
#> 93     5  1054 1055
#> 94     7  1064 1065
#> 95     4  1071 1071
#> 96    40  1074 1093
#> 
#> $RecordTime
#>   [1]    0    1    2    3    7    9   10   12   13   14   18   19   23   24   25
#>  [16]   26   27   28   29   32   33   34   41   42   43   44   49   50   53   55
#>  [31]   58   59   60   61   62   63   64   65   69   70   71   74   75   76   79
#>  [46]   85   86   87   88   89   92  249  251  252  253  254  267  268  269  270
#>  [61]  271  274  275  276  278  285  286  291  293  294  299  303  304  305  306
#>  [76]  308  309  330  331  332  333  334  335  336  339  343  344  345  349  355
#>  [91]  356  357  358  359  363  364  374  375  376  377  378  381  385  386  389
#> [106]  391  401  402  405  406  407  408  409  416  417  419  420  421  422  424
#> [121]  425  438  442  443  444  445  446  455  456  459  460  463  464  465  489
#> [136]  490  492  512  515  517  527  530  533  534  535  536  537  538  543  544
#> [151]  545  548  565  566  574  576  577  582  583  598  607  717  718  719  721
#> [166]  722  723  731  734  745  746  747  748  752  819  829  840  843  844  845
#> [181]  849  850  894  896  898  920  921  922  924  931  934  935  941  942  943
#> [196]  975  977  978  979  982  983  984  985  986  987  988  991  992  994 1043
#> [211] 1052 1053 1057 1058 1059 1060 1061 1062 1063 1067 1068 1069 1070
head(sort(localScoreC(LongSeqScore.inv)$suboptimalSegmentScores[,1], decreasing = TRUE))
#> [1] 65 40 36 33 25 23
plot(1:1093,lindley(LongSeqScore.inv), type = 'l')
```



That leads to: first excursion equal to 6; last excursion, the 96th one, equal to 38.

The corresponding *p*-values are:

```
markov_parameters <- sequences2transmatrix(LongSeqScore.inv)
lambda.inv <- markov_parameters$transition_matrix
system.time(pv5<-proba_theoretical_ith_excursion_markov(a = 38, theta,
                                            lambda.inv,
                                            score_function, i = 96
                                            )$proba_q_i_geq_a)
#> utilisateur     système      écoulé
#>      35.218       0.017      35.267
pv5
#> [1] 0.005460216
proba_theoretical_ith_excursion_markov(a = 6, theta, lambda.inv,
                                            score_function,i = 1
                                            )$proba_q_i_geq_a
#> [1] 0.1765172
```

Last excursion, the 96th one, equal to 38, with *p*-value equal to 0.53%. First excursion equal to 6, with *p*-value equal to 17%.

The last excursion is still significant. The first one is still non significant.

Remark : Even with a Bonferroni correction to take into account the multiple test (here two studies excursions), the sequence possesses a significant segment. Whereas considering the highest value over all the excursions of the whole sequence, the local score value, the sequence is not significant.

What about the excursion realising the local score

Let us consider the excursion 30 as an excursion among the others. For the first way to read the protein:

```
a <- 65
i <- 30
proba_theoretical_ith_excursion_markov(a, theta,
                                       lambda, score_function,
                                       i)$proba_q_i_geq_a
#> [1] 0.0004221151
```

There is a less than 4 in 10,000 chance of having a first mountain over 65.

Or with the second way to read the protein:

```
subOptSegment.inv <- localScoreC(LongSeqScore.inv)$suboptimalSegmentScores
which.max(subOptSegment.inv[,1])
#> [1] 16
print(subOptSegment.inv[16,])
#>    value begin end
#> 16    65    93 138

a <- 65
i <- 16
proba_theoretical_ith_excursion_markov(a, theta, lambda.inv,
                                       score_function,i)$proba_q_i_geq_a
#> [1] 0.0004379929
```

As expected we obtain a similar *p*-value.

Although the highest mountain of height 65 in this sequence of length 1093 does not have a significant value at the 5% threshold, it does not mean that the sequence has not interesting segments. Observing a mountain exceeding 40 is significant for example, and it is the same for values 36 and 33. All these tests remain significant even if a correction of type Bonferroni is taken.