Package 'hydra'

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Type Package

Title Hyperbolic Embedding

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Description Calculate an optimal embedding of a set of data points into low-
dimensional hyperbolic space. This uses the strain-minimizing hyperbolic embedding of Keller-
Ressel and Nargang (2019), see <arXiv:1903.08977>.

Suggests igraph, igraphdata, Matrix, RSpectra

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R topics documented:

| hydra | *Calculate hyperbolic embedding of distance data* |

Description

Implements the HYDRA (hyperbolic distance recovery and approximation) method for embedding high-dimensional data points (represented by their distance matrix D) into low-dimensional hyperbolic space.

Usage

```
hydra(D, dim = 2, curvature = 1, alpha = 1.1, equi.adj = 0.5,
  control = list())
```

Arguments

D	a square symmetric matrix of distances (or dissimiliarities) to be embdedded, can also be a [dist](#) object
dim	embedding dimension
curvature	embedding curvature; if this argument is NULL, hydra tries to find the optimal curvature
alpha	real number greater one; adjusts the hyperbolic curvature. Values larger than one yield a more distorted embedding where points are pushed to the outer boundary (i.e. the ideal points) of hyperblic space. The interaction between curvature and alpha is non-linear.
equi.adj	equi-angular adjustment; must be a real number between zero and one; only used if dim is 2. Value 0 means no ajustment, 1 adjusts embedded data points such that their angular coordinates in the Poincare disc are uniformly distributed. Other values interpolate between the two extremes. Setting the parameter to non-zero values can make the embedding result look more harmoniuous in plots.
control	a list which may contain the following boolean flags:

- polar - return polar coordinates in dimension 2 (default: TRUE if dim is 2. This flag is ignored in higher dimension)
- isotropic.adj - perform isotropic adjustment, ignoring Eigenvalues (default: TRUE if dim is 2, FALSE else)
- return.lorentz - return raw Lorentz coordinates (before projection to hyperbolic space) (default: FALSE)
- return.stress - return embedding stress (default: TRUE)
- return.dist - return hyperbolic distance matrix of embedded points (default: FALSE)
- use.eigs - use [eigs](#) function from **RSpectra** and [norm](#) function from **Matrix** to speed up computation (default: FALSE)

Details

See https://arxiv.org/abs/1903.08977 for more details.

Value

A 'hydra' object, which is a list with all or some of the following components:

r a vector containing the radial coordinates of the embedded points

directional a matrix with `dim` columns containing as rows the directional coordinates of the embedded points

theta a vector containing the angular coordinates of the embedded points (only returned if `dim` is 2 and `polar` flag is TRUE)

curvature the curvature used for the returned embedding

dim the dimension used for the returned embedding

stress the stress (i.e. the mean-square difference) between distances supplied in `D` and the hyperbolic distance matrix of the returned embedding

dist the hyperbolic distance matrix of the returned embedding (only returned if flag `return.dist` is true. Computation may be time- and memory-intensive.)

x0 a vector containing the 'time-like' coordinate of the raw Lorentz embedding (only returned if flag `return.lorentz` is true)

X a matrix with `dim` columns containing as rows the 'space-like' coordinate of the raw Lorentz embedding (only returned if flag `return.lorentz` is true)

Author(s)

Martin Keller-Ressel <martin.keller-ressel@tu-dresden.de>

Examples

```
data(karate)
embedding <- hydra(karate$distance)
plot(embedding,labels=karate$label,lab.col=karate$group,graph.adj=karate$adjacency)

## Compare with Multidimensional scaling (MDS):
mds <- cmdscale(karate$distance) # Compute Euclidean embedding with MDS
mds.stress <- sqrt(sum((as.matrix(dist(mds)) - karate$distance)^2)) # Calculate embedding stress
c(embedding$stress,mds.stress) # Compare hyperbolic with Euclidean stress
```

| hydraPlus | [hydra](#) *with additional stress minimization* |

Description

Runs the [hydra](#) method and then performs a further optimization step by minimizing the stress of the embedding and optimizing hyperbolic curvature

Usage

```
hydraPlus(D, dim = 2, curvature = 1, alpha = 1.1, equi.adj = 0.5,
  control = list(), curvature.bias = 1, curvature.freeze = TRUE,
  curvature.max = NULL, maxit = 1000, ...)
```

Arguments

D	a square symmetric matrix of distances (or dissimiliarities) to be embdedded, can also be a [dist](#) object
dim	embedding dimension
curvature	embedding curvature; if this argument is NULL, hydra tries to find the optimal curvature
alpha	real number greater one; adjusts the hyperbolic curvature. Values larger than one yield a more distorted embedding where points are pushed to the outer boundary (i.e. the ideal points) of hyperblic space. The interaction between curvature and alpha is non-linear.
equi.adj	equi-angular adjustment; must be a real number between zero and one; only used if dim is 2. Value 0 means no ajustment, 1 adjusts embedded data points such that their angular coordinates in the Poincare disc are uniformly distributed. Other values interpolate between the two extremes. Setting the parameter to non-zero values can make the embedding result look more harmoniuous in plots.
control	a list which may contain the following boolean flags:

     • polar - return polar coordinates in dimension 2 (default: TRUE if dim is 2. This flag is ignored in higher dimension)

     • isotropic.adj - perform isotropic adjustment, ignoring Eigenvalues (default: TRUE if dim is 2, FALSE else)

     • return.lorentz - return raw Lorentz coordinates (before projection to hyperbolic space) (default: FALSE)

     • return.stress - return embedding stress (default: TRUE)

     • return.dist - return hyperbolic distance matrix of embedded points (default: FALSE)

     • use.eigs - use [eigs](#) function from **RSpectra** and [norm](#) function from **Matrix** to speed up computation (default: FALSE)

curvature.bias	Modify curvature before stress minimization by multiplying with curvature.bias

curvature.freeze

    Freeze the curvature returned by [hydra](). If TRUE then no optimization of curvature is attempted in the second stage of the algorithm. If FALSE then curvature is optimized in the second stage

curvature.max   Upper bound for the curvature. If NULL, a defulat bound is used

maxit   Maximal number of iterations. This parameter is passed to the optimization routine [optim]()

...   Additional parameters are passed to [optim](), which performs the underlying stress minimization

Details

See https://arxiv.org/abs/1903.08977 for more details.

Value

A 'hydra' object, which is a list with all or some of the following components:

r a vector containing the radial coordinates of the embedded points

directional a matrix with `dim` columns containing as rows the directional coordinates of the embedded points

theta a vector containing the angular coordinates of the embedded points (only returned if `dim` is 2 and `polar` flag is TRUE)

curvature the curvature used for the returned embedding

dim the dimension used for the returned embedding

stress the stress (i.e. the mean-square difference) between distances supplied in `D` and the hyperbolic distance matrix of the returned embedding

dist the hyperbolic distance matrix of the returned embedding (only returned if flag `return.dist` is true. Computation may be time- and memory-intensive.)

x0 a vector containing the 'time-like' coordinate of the raw Lorentz embedding (only returned if flag `return.lorentz` is true)

X a matrix with `dim` columns containing as rows the 'space-like' coordinate of the raw Lorentz embedding (only returned if flag `return.lorentz` is true)

Author(s)

Martin Keller-Ressel <martin.keller-ressel@tu-dresden.de>

Examples

```
data(karate)
embedding <- hydraPlus(karate$distance)
plot(embedding,labels=karate$label,node.col=karate$group,graph.adj=karate$adjacency)
```

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karate *Zachary's karate club network data*

Description

The social network of a university karate club described in the paper "An Information Flow Model for Conflict and Fission in Small Groups" by Wayne W. Zachary.

Edges indicate social interactions between members

Usage

```
data(karate)
```

Format

A list with the elements:

adjacency The adjacency matrix of the karate club network

distances The shortest-path distance between nodes

labels Node labels. 'A' and 'H' are John A. and Mr. Hi, who led the two groups after the split of the club

group The two groups after the split

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plot.hydra *Plot a hyperbolic embedding*

Description

Plot a two-dimensional hyperbolic embedding as returned by hydra in the Poincare disc

Usage

```
## S3 method for class 'hydra'
plot(x, labels = NULL, node.col = 1, pch = NULL,
  graph.adj = NULL, crop.disc = TRUE, shrink.disc = FALSE,
  disc.col = "grey90", rotation = 0, mark.center = 0,
  mark.angles = 0, mildify = 3, cex = 1, ...)
```

Arguments

x	a hydra object as returned by [hydra](#) with dimension `dim` equal 2
labels	character labels for the embedded points, supplied as a vector. NULL triggers default values
node.col	colors for the labels and/or points, supplied as a vector. NULL triggers default values. See 'Color Specification' in [par](#) for details
pch	plotting 'characters' for the embedded points. supplied as a vector. NULL triggers default values. See [points](#) for details
graph.adj	a graph adjacency matrix that is used to plot links between the embedded points (links are drawn for all non-zero elements of `graph.adj`)
crop.disc	should the Poincare disc be cropped or fully shown? Defaults to TRUE
shrink.disc	if true, the Poincare disc is shrunk to tightly fit all plotted points. Defaults to FALSE
disc.col	color of the Poincare disc. Set to "white" to hide disc
rotation	rotate points by this angle (specified in degrees) around the center of the Poincare disc
mark.center	Should a cross be placed at the center of the disc? If 0, nothing is drawn. Other values specify the relative size of the cross mark.
mark.angles	Should the angular coordinates of points be marked at the boundary of the disc? If 0, nothing is drawn. Other values specify the relative size of the angle marks.
mildify	large values reduce the curvature of links. Values around 3 are visually most appealing. Setting `mildify` to 1 shows the true hyperbolic curvature
cex	character expansion for labels and points, supplied as a numerical vector. See also [points](#)
...	all other parameters are passed on as additional graphical parameters (see [par](#))

Author(s)

Martin Keller-Ressel <martin.keller-ressel@tu-dresden.de>

Examples

```
data(karate)
embedding <- hydra(karate$distance)
plot(embedding,labels=karate$label,node.col=karate$group,graph.adj=karate$adjacency)

# plot points instead of labels, hide Poincare disc and rotate by 90 degrees:
plot(embedding,pch=karate$group, node.col=karate$group,graph.adj=karate$adjacency, disc.col="white",
     rotation=90)

# do not crop the Poincare disc, mark the center and mark angles:
plot(embedding,labels=karate$label, node.col=karate$group,graph.adj=karate$adjacency,
     crop.disc=FALSE, mark.center=0.05, mark.angles=0.025)
```

Index